

Many-Body Approach to Mesons, Hybrids and Glueballs

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We represent QCD at the hadronic scale by means of an effective Hamiltonian, H , formulated in the Coulomb gauge. As in the Nambu-Jona-Lasinio model, chiral symmetry is dynamically broken, however our approach is renormalizable and also includes confinement through a linear potential with slope specified by lattice gauge theory. We perform a comparative study of alternative many-body techniques for approximately diagonalizing H : BCS for the vacuum ground state; TDA and RPA for the excited hadron states. We adequately describe the experimental meson and lattice glueball spectra and perform the first relativistic, three quasiparticle calculation for hybrid mesons. In general agreement with alternative theoretical approaches, we predict the lightest hybrid states near but above 2 GeV , indicating the two recently observed $J^{PC} = 1^{-+}$ exotics at 1.4 and 1.6 GeV are of a different, perhaps four quark, structure. We also detail a new isospin dependent interaction from $q\bar{q}$ color octet annihilation (analogous to ortho positronium) which splits $I = 0$ and $I = 1$ states.

1 Introduction and Model Hamiltonian in the Coulomb Gauge

In the past several years we have implemented^{1,2,3,4} an ambitious QCD program to comprehensively investigate hadron structure. Central to this project is a realistic effective Hamiltonian which we develop from QCD through renormalization and subsequently diagonalize using established many-body techniques. We are particularly interested in the signature QCD prediction of exotic states (hadrons with quantum numbers not possible in pure $q\bar{q}$ or qqq models). Early evidence⁵ for the existence of an exotic hadron with $J^{PC} = 1^{-+}$, denoted $\hat{\rho}$, has finally been confirmed with the recent BNL observation⁶ of two states at 1.4 and 1.6 GeV . These measurements, along with anticipated future hybrid experiments (COMPASS at CERN and Hall D at JLab), have motivated the study detailed in this paper.

The theoretical situation is not satisfactory: lattice results⁷ indicate that the lightest reliable hybrid meson mass with quantum numbers 1^{-+} should appear at $(2.0 \pm 0.2)\text{ MeV}$. On the other hand, vintage bag model results⁸ and a recent QCD sum rule calculation⁹ agree in predicting exotic masses, between 1.4 and 1.7 GeV . The sum rule prediction⁹ claims a rigorous 1.9 GeV upper bound which is clearly below the lattice result. The lattice calculations, however, do require extrapolating for the u, d sector and are therefore less accurate for the light quark hadrons. Hence, further insight is definitely needed to determine if these exotic states are predominantly $q\bar{q}g$ objects or belong to other Fock space sectors, such as $q\bar{q}q\bar{q}$.

Our many-body Hamiltonian approach, which has been successfully tested for both glueballs¹ and conventional mesons⁴, addresses this issue. Before treating the 3-body hybrid meson spectrum we present our model Hamiltonian in detail and briefly summarize our previous results. Our QCD inspired, effective Hamiltonian is formulated in the Coulomb gauge¹⁰

$$H = \int d\mathbf{x} \Psi^\dagger(\mathbf{x}) (-i\alpha \cdot \nabla + \beta m) \Psi(\mathbf{x}) + Tr \int d\mathbf{x} (\boldsymbol{\Pi}^a \cdot \boldsymbol{\Pi}^a + \mathbf{B}_A^a \cdot \mathbf{B}_A^a) - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \rho^a(\mathbf{x}) V(|\mathbf{x} - \mathbf{y}|) \rho^a(\mathbf{y}) , \quad (1)$$

containing both quark, Ψ , and gluon, \mathbf{A}^a , $\mathbf{B}_A^a = \nabla \times \mathbf{A}^a$, fields with color density $\rho^a = \Psi^\dagger \frac{\lambda^a}{2} \Psi + f^{abc} \mathbf{A}^b \cdot \boldsymbol{\Pi}^c$. We adopt the standard current quark mass, m , values for the u, d, s and c flavors; $m_u = m_d = 5 \text{ MeV}$, $m_s = 150 \text{ MeV}$ and $m_c = 1200 \text{ MeV}$. The linear interaction, $V_L = \sigma r$, is obtained from lattice measurements and Regge phenomenology yielding $\sigma = 0.18 \text{ GeV}^2$. For certain observables we supplement this with the canonical Coulomb potential $V_C = -\frac{\alpha_s}{r}$ ($\alpha_s = \frac{g_s^2}{4\pi} \approx 0.2 - 0.4$). The normal mode field expansions are

$$\begin{aligned} A_i^a(\mathbf{x}) &= \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} [a_i^a(\mathbf{k}) + a_i^{a\dagger}(-\mathbf{k})] e^{i\mathbf{k} \cdot \mathbf{x}} \\ \Pi_i^a(\mathbf{x}) &= -i \int \frac{d\mathbf{k}}{(2\pi)^3} \sqrt{\frac{\omega_k}{2}} [a_i^a(\mathbf{k}) - a_i^{a\dagger}(-\mathbf{k})] e^{i\mathbf{k} \cdot \mathbf{x}} \\ \Psi(\mathbf{x}) &= \sum_{c\lambda} \int \frac{d\mathbf{k}}{(2\pi)^3} \left[u_{c\lambda}(\mathbf{k}) b_{c\lambda}(\mathbf{k}) + v_{c\lambda}(-\mathbf{k}) d_{c\lambda}^\dagger(-\mathbf{k}) \right] e^{i\mathbf{k} \cdot \mathbf{x}} , \end{aligned} \quad (2)$$

and with the Coulomb gauge transversality condition, $\mathbf{k} \cdot \mathbf{a}^a(\mathbf{k}) = 0$, yields the commutation relation

$$[a_i^a(\mathbf{k}), a_j^{b\dagger}(\mathbf{q})] = \delta^{ab} (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{q}) (\delta_{ij} - \hat{k}_i \hat{k}_j) . \quad (3)$$

We note the color density-density interaction, $\rho^a(\mathbf{x}) V(|\mathbf{x} - \mathbf{y}|) \rho^a(\mathbf{y})$, only produces color singlet states in the spectrum^{4,11,12} when V is a linear, harmonic oscillator or any Fourier transformed confining interaction with a singular behavior as $q \rightarrow k$. This is a stringent check in our calculations, since the matrix elements must always contain factors of $q - k$ to cancel this singularity. In particular, for our chosen linear potential, we have in momentum space $\hat{V}(|\mathbf{q} - \mathbf{k}|) = -\frac{8\pi\sigma}{(q-k)^4}$.

2 Ground State and BCS Mass Gap Equations

We now wish to solve $H\Psi = E\Psi$ as accurately as possible. In this section we focus on the ground state and introduce the Bardeen, Cooper and Schrieffer (BCS) transformation. We proceed by normal-ordering the Hamiltonian using the basis, Eq. (2), and minimize the ground state, or vacuum, expectation value. The key concept in this approach is that of a quasiparticle. The bare, current operators a , b , d are rotated to improved quasiparticle operators α , B , D by means of the BCS transformation. The vacuum state, $|\Omega\rangle$, is determined by the relations $B|\Omega\rangle = D|\Omega\rangle = \alpha|\Omega\rangle = 0$. The gluon and quark BCS rotations are

$$\begin{aligned}\alpha_i^a(\mathbf{k}) &= \cosh \Theta_k a_i^a(\mathbf{k}) + \sinh \Theta_k a_i^{a\dagger}(-\mathbf{k}) \\ B_{c\lambda}(\mathbf{k}) &= \cos \frac{\theta_k}{2} b_{c\lambda}(\mathbf{k}) - \lambda \sin \frac{\theta_k}{2} d_{c\lambda}^\dagger(-\mathbf{k}) \\ D_{c\lambda}(-\mathbf{k}) &= \cos \frac{\theta_k}{2} d_{c\lambda}(-\mathbf{k}) + \lambda \sin \frac{\theta_k}{2} b_{c\lambda}^\dagger(\mathbf{k}) ,\end{aligned}\tag{4}$$

where Θ_k , $\theta_k/2$ are the BCS angles, further specified below. This also introduces a redefinition of the spectral function, ω_k , and counter-rotated quasiparticle spinors U , V such that the expansions of the fields remain formally invariant.

The specific variational parameters are the quark gap angle, ϕ_k , related to the BCS angle by $\tan(\phi_k - \theta_k) = m/k$, and the gluon self-energy, ω_k , satisfying $\omega_k = ke^{-2\Theta_k}$. The vacuum contains correlated Cooper pairs and explicitly breaks chiral symmetry due to the form of ω and ϕ ^{4,12,13}. The functional variation with respect to the two parameters θ_k , Θ_k

$$\delta \left(\frac{\langle \Omega | H | \Omega \rangle}{\langle \Omega | \Omega \rangle} \right) = 0$$

generates two integral (mass gap) equations for the quark and gluon sectors, respectively

$$k \sin \phi_k - m \cos \phi_k = \frac{2}{3} \int \frac{d\mathbf{q}}{(2\pi)^3} \hat{V}(|\mathbf{k} - \mathbf{q}|) [\sin \phi_k \cos \phi_q \hat{\mathbf{k}} \cdot \hat{\mathbf{q}} - \sin \phi_q \cos \phi_k] \tag{5}$$

$$\omega_k^2 = k^2 - \frac{3}{4} \int \frac{d\mathbf{q}}{(2\pi)^3} \hat{V}(|\mathbf{k} - \mathbf{q}|) (1 + (\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})^2) \left(\frac{w_q^2 - w_k^2}{w_q} \right) . \tag{6}$$

Significantly, the BCS vacuum is stable against quasiparticle pair creation since the "anomalous" terms in the Hamiltonian of the type $\alpha\alpha$, $\alpha^\dagger\alpha^\dagger$, BD , $B^\dagger D^\dagger$ are also cancelled by the same rotation.

For the linear potential $1/q^4$, simple dimensional analysis reveals Eq. (5) is UV finite while Eq. (6) is logarithmically divergent. Hence we impose a momentum cutoff $\Lambda = 4 \text{ GeV}$ and proceed to solve both numerically after performing a standard three dimensional reduction. The details of this calculation are given elsewhere⁴. We verify that our variational solution has a minimum in energy (traditional Mexican hat shape).

3 TDA and RPA for Mesons and Glueballs

With these quasiparticle degrees of freedom we now construct the excited states from this vacuum. For both the quark and gluon sector we represent hadrons as quasiparticle pairs (eg. $q\bar{q}$ for mesons and gg for glueballs) and angular momenta couple to form states of good J^{PC} . Next we invoke the TDA at the 1p-1h level and diagonalize the model Hamiltonian in this truncated space. A subset of our extensive TDA calculations is displayed in Fig. 1 for the pseudoscalar mesons. In general there is broad agreement with the data, the most notable exception being for the π and η .

The insufficient $\approx 200 \text{ MeV}$, π/ρ mass splitting and related issue of chiral symmetry motivated our improved RPA treatment. Now the pion creation operator generalizes to

$$Q^\dagger = \sum_{ij} \left(X_{ij} q_i^\dagger \bar{q}_j^\dagger - Y_{ij} q_i \bar{q}_j \right),$$

with pion state $|\pi\rangle = Q^\dagger |RPA\rangle$ and improved vacuum satisfying $Q |RPA\rangle = 0$. Here q_i , \bar{q}_i are again the BCS rotated quasiparticle operators and X_{ij} , Y_{ij} are the RPA wavefunction components obtained from the coupled equations of motions generated by

$$\langle \pi | [H, Q^\dagger] | RPA \rangle = M_\pi \langle \pi | Q^\dagger | RPA \rangle. \quad (7)$$

In the chiral limit ($m = 0$) the chiral condensate operator commutes with Q^\dagger and we rigorously compute $M_\pi = 0$ consistent with Goldstone's theorem. For $m = 5 \text{ MeV}$ explicit chiral symmetry breaking yields $M_\pi = 294 \text{ MeV}$, significantly better than the TDA value (note, the physical pion mass is reproduced with $m = 3 \text{ MeV}$). Since both TDA and RPA produce comparable spin splittings, we therefore conclude that chiral symmetry, which only the RPA respects, is responsible for most of the π/ρ mass difference.

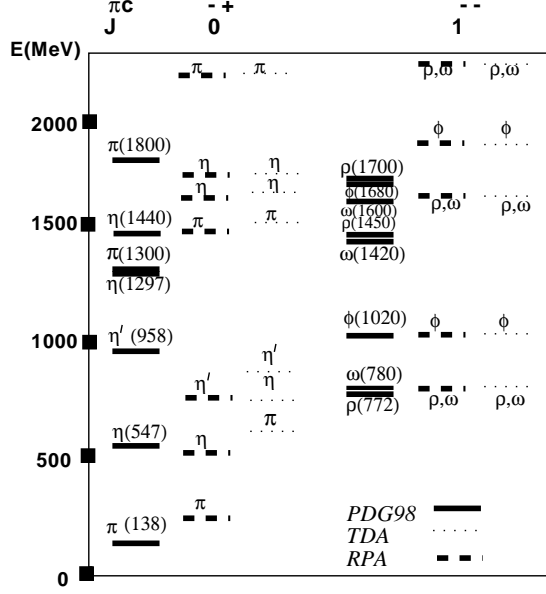


Figure 1: TDA (dots), RPA (dashes) and data (bars) for the light pseudoscalar and vector meson spectra.

Similarly, we generate the glueball spectrum for two quasiparticle gluons. Now chiral symmetry is not an issue and the RPA and TDA spectra agree to within a few percent. Our calculations, using the same string tension as above, reproduce the lattice measurements as discussed in Confinement II.

A fundamental test for QCD is the existence of exotic mesons (quantum numbers not possible in any $q\bar{q}$ model). In particular, it has been speculated that two recently observed⁶ states with isospin 1 and $J^{PC} = 1^{-+}$ at 1.4 and 1.6 GeV contain explicit glue. Of several gluonic scenarios, glueballs (oddballs) must be eliminated since they have isospin 0. We therefore investigate in the next section if these 1^{-+} states are hybrids.

4 Three-Body TDA for Hybrid Mesons

We formulate the hybrid meson as a 3-body problem ($q\bar{q}g$) with TDA color wavefunction

$$|hybrid\rangle = B^\dagger D^\dagger \alpha^\dagger |BCS\rangle \equiv [(B^\dagger \otimes D^\dagger)_8 \otimes \alpha^\dagger]_0 |BCS\rangle,$$

where the three BCS quasiparticles have momenta \mathbf{q} , $\bar{\mathbf{q}}$, and \mathbf{g} referred to the hybrid cm frame. The optimal choice of relative variables is $\mathbf{q}_+ = \frac{\mathbf{q} + \bar{\mathbf{q}}}{2}$,

$\mathbf{q}_- = \mathbf{q} - \bar{\mathbf{q}}$ which facilitates implementing rotational, parity (P) and C -parity symmetries. The complete TDA wavefunction ansatz with spin (λ) and color (c, a) indices is

$$|hybrid\rangle = \sum_{\lambda_q \lambda_{\bar{q}} \lambda_{cc\bar{a}}} \iint \frac{d\mathbf{q}_-}{(2\pi)^3} \frac{d\mathbf{q}_+}{(2\pi)^3} F_{\lambda_q \lambda_{\bar{q}} \lambda}^{JPC}(\mathbf{q}_+, \mathbf{q}_-) T_{c\bar{c}}^a B_{\lambda_q c}^\dagger(\mathbf{q}) D_{\lambda_{\bar{q}} \bar{c}}^\dagger(\bar{\mathbf{q}}) \alpha_\lambda^{a\dagger}(\mathbf{g}) |\Omega\rangle .$$

This wavefunction's rotation properties are governed by the three tensor indices $\lambda_q, \lambda_{\bar{q}}, \lambda$, and the two arguments $\mathbf{q}_+, \mathbf{q}_-$. To construct an appropriate $SU(2)$ representation we need to couple five angular momenta (three quasiparticle intrinsic spins and two orbital, L_\pm , associated with \mathbf{q}_\pm) to give a total J, m_J . To accomodate the Coulomb gauge transversality condition we utilize a modified LS coupling scheme involving two intermediate angular momenta $\mathbf{l} = \mathbf{L}_+ + \mathbf{l}$ and $\mathbf{L} = \mathbf{l} + \mathbf{L}_-$. The resulting angular momentum decomposition is

$$\begin{aligned} F_{\lambda_g \lambda_q \lambda_{\bar{q}}}^{JPC}(\mathbf{q}_+, \mathbf{q}_-) = & \sum_{lL-L_+LSm_+m_-} F_{lL-L_+LS}^{JPC}(|\mathbf{q}_+|, |\mathbf{q}_-|) Y_{L_+}^{m_+}(\hat{\mathbf{q}}_+) Y_{L_-}^{m_-}(\hat{\mathbf{q}}_-) \\ & (-1)^{\lambda_g} \langle L_+ m_+ 1 - \lambda_g | l m_l \rangle \langle L_- m_- l m_l | L m_L \rangle \\ & \langle \frac{1}{2} \lambda_q \frac{1}{2} - \lambda_{\bar{q}} | S m_S \rangle (-1)^{\frac{1}{2} - \lambda_{\bar{q}}} \langle L m_L S m_S | J m_J \rangle . \end{aligned}$$

Due to the transversality condition mentioned above, $\hat{\mathbf{q}}_+ \cdot \alpha = 0$, so

$$\langle 1 m_+ 1 \lambda | 00 \rangle Y_1^{m_+}(\hat{\mathbf{q}}_+) \alpha_\lambda^\dagger = 0 \quad (8)$$

and therefore we cannot couple the angular momenta of the gluon to give the intermediate state $|l m_l\rangle = |00\rangle$.

With this coupling scheme the total parity is a product of the intrinsic parities of the quark-antiquark pair (-1), the intrinsic parity of the gluon (-1), and the two spherical harmonics yielding

$$P = (-1) \cdot (-1) \cdot (-1)^{L_+ + L_-} = (-1)^{L_+ + L_-} .$$

Similarly the total charge conjugation is a product of the quark-antiquark pair $(-1)^{L_+ + S}$ from equivalence to the exchange of all $q\bar{q}$ quantum numbers) and the gluon (-1), which is its own antiparticle and has odd C -parity. Hence

$$C = (-1) \cdot (-1)^{L_- + S} = (-1)^{1 + L_- + S} .$$

Notice the extra C -parity sign which now permits exotic quantum numbers. We are only interested in the lightest hybrids which will have S and P waves

Table 1: Possible Quantum Numbers for a $q\bar{q}g$ Hybrid Meson

L_+	L_-	S	L	J	P	C	
0	0	0	1	1	+	-	
0	0	1	1	0	+	+	
0	0	1	1	1	+	+	
0	0	1	1	2	+	+	
0	1	0	0	0	-	+	
0	1	0	1	1	-	+	Exotic
0	1	0	2	2	-	+	
0	1	1	0	1	-	-	
0	1	1	1	0	-	-	Exotic
0	1	1	1	1	-	-	
0	1	1	1	2	-	-	
0	1	1	2	1	-	-	
0	1	1	2	2	-	-	
0	1	1	2	3	-	-	
1	0	0	0	0	-	-	Forbidden by transversality
1	0	0	1	1	-	-	
1	0	0	2	2	-	-	
1	0	1	0	1	-	+	Forbidden by transversality
1	0	1	1	0	-	+	
1	0	1	1	1	-	+	Exotic
1	0	1	1	2	-	+	
1	0	1	2	1	-	+	Exotic
1	0	1	2	2	-	+	
1	0	1	2	3	-	+	Exotic

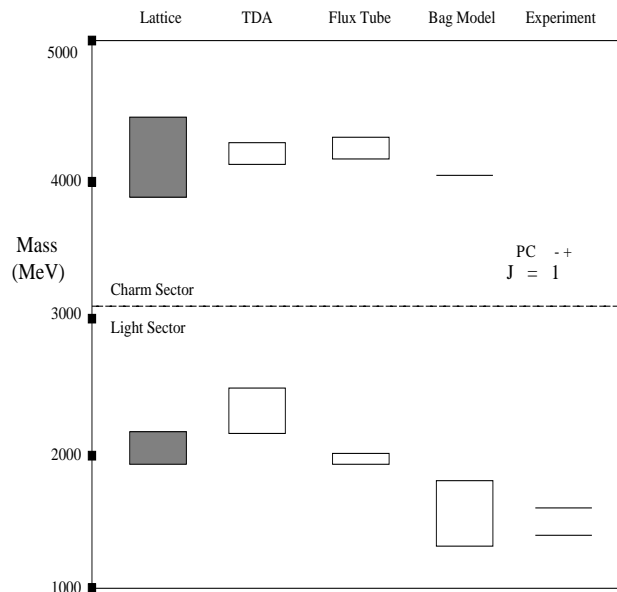


Figure 2: Light and charmed exotic 1^{-+} hybrid mesons. The TDA calculation is comparable to the lattice results but both disagree with observed resonances.

for either of the angular momenta L_+ or L_- . The resulting possible quantum numbers are displayed in Table 1.

In principle all the states with the same J^{PC} quantum numbers can mix but we do not address this issue here. We also note that our wavefunction normalization is not standard due to the transversality relation, Eq. (3), which introduces an extra angular momentum coefficient.

Finally, in analogy with our two-body TDA treatment the hybrid equation of motion is

$$\langle q\bar{q}g|H|hybrid\rangle = E\langle q\bar{q}g|hybrid\rangle . \quad (9)$$

Using Eqs. (1, 2) and the hybrid wavefunction expansion, we obtain the TDA equation of motion which now is a non-local equation in the two variables \mathbf{q}_+ , \mathbf{q}_- (12-dimensional problem) precluding practical matrix diagonalization. Therefore we utilize alternative ansätze for the radial wavefunction and variationally evaluate the Hamiltonian matrix element.

Separating the cm variables, the integrals can be reduced to 9-dimensions which we evaluated by Monte Carlo methods utilizing Lepage's code VEGAS. We employed an increasing number of points, up to several million, until sat-

isfactory convergence was achieved. The angular wavefunctions are explicitly coded in terms of spherical harmonics with all magnetic spin sums and angular integrals evaluated numerically. The integrable IR singularity from our potential received special attention. We divided the complete integral into different parts depending upon the Hamiltonian components and then used a change of variables, always placing the singularity at the origin, with a Jacobian transformation to concentrate points there. This is detailed in our previous work.

In Fig. 2 we present the main result of this work, our prediction of the light and charmed exotic hybrid mass. We also compare to alternative lattice, flux tube and even vintage bag model approaches as well as the recent E852 Brookhaven data. Notice that, with the exception of the dated bag model results, the different theoretical models generally agree but disagree with observation.

Because of the potential ramifications of this disagreement between measurement and predictions we further examined and tested our model and calculation in detail. First, to ensure our variational method is accurate, we compared several different radial trial wavefunctions. In particular, we used gaussian and also more complicated functions generated by the exact numerical solutions to simpler, 2-body problems (ρ meson, J/Ψ , glueball) with various potentials. We also added nodes as needed. We found the maximum variance (sensitivity) to wavefunction choice to be about 50 MeV in the hybrid mass. Second, we also considered whether chiral symmetry affects the hybrid meson spectrum and variationally formulated the RPA as above, but now with an additional constituent TDA gluon. Interestingly, we find RPA and TDA to be numerically equivalent. The reason is that the quarks are now in a color octet state but the chiral octet charge

$$Q_5^a = \int d\mathbf{x} \Psi^\dagger(\mathbf{x}) \gamma_5 T^a \Psi(\mathbf{x})$$

is not conserved since the commutator with the Hamiltonian does not vanish. Further, there is also no Gell-Mann-Oakes-Renner relation and no Goldstone hybrid boson in the chiral limit $m \rightarrow 0$. Therefore we do not expect any major numerical effect in the hybrid spectrum, analogous to the 400 MeV downward shift of the chiral pion. This was confirmed by an explicit numerical calculation in a selected channel.

Consequently, we are reasonably confident in our prediction that the lightest exotic hybrid mass is above 2 GeV and concur with other contemporary theoretical model results. This also strongly indicates that the observed exotic 1^{-+} at 1.4 and 1.6 GeV are not hybrid mesons but have an alternative, four quark or meson molecular structure.

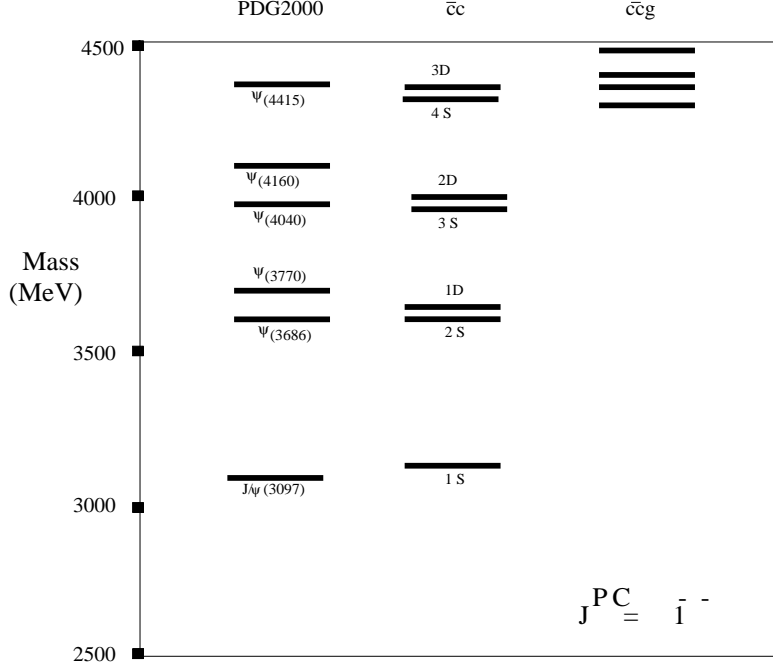


Figure 3: TDA theory for conventional ($c\bar{c}$) and non-exotic ($c\bar{c}g$) states compared to the observed 1^{--} J/ψ spectrum¹⁴.

Our last key result concerns the charmonium J/Ψ spectrum and is shown in Fig. 3. By simply calculating D wave TDA $c\bar{c}$ states we have resolved the historic "overpopulation" problem. Ironically, if we then include our predicted non-exotic charmonium states, also shown in the figure, we now have an "underpopulation" problem which is more typical of confrontation between the conventional quark model and experiment. It would now appear that simply counting states may not be that effective in identifying gluon rich states.

Finally, since any three quasiparticle problem can be treated variationally, we have explicitly constructed valence baryons (qqq) states and evaluated both the nucleon (N) and $\Delta_{3/2}$ with our same Hamiltonian. Our Δ mass is in agreement with data, but only about 1/3 of the N - Δ mass splitting is obtained. The other interesting case is the triple gluon glueball, ggg , whose study is still preliminary and will be reported in the near future. This exhausts the possible three body states which can form color singlets.

5 Conclusions

In summary, the BCS, TDA and RPA many-body treatments are powerful, effective methods for investigating hadron structure. Using a relativistic, field theoretical Hamiltonian with standard quark masses and only one (pre-determined) interaction parameter, our approximate many-body solutions yield reasonable descriptions of the vacuum, mesons and glueballs. In particular, the RPA properly incorporates chiral symmetry and provides new insight into the condensate structure of the vacuum and chiral governance of the pion. Most significantly, our hybrid meson mass prediction is above 2 GeV and in reasonable agreement with both lattice and flux tube results. This strongly suggests that the recent observed exotic states below 2 GeV are not hybrids but more likely 4 quark states. Because our method is directly amenable to including higher Fock state components, we are currently calculating the molecular ($q\bar{q}q\bar{q}$) meson spectrum. Future work will also entail extensions to the nucleon strangeness content (pentaquark systems) and even dibaryons (six quark systems) which are both tractable in our approach.

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